Journal of Engineering Physics and Thermophysics, Vol. 74, No. 1, 2001

SIMULATION OF COOLING OF WATER DROPLET AND FILM FLOWS IN LARGE NATURAL WET COOLING TOWERS

A. I. Petruchik, A. D. Solodukhin, and S. P. Fisenko

UDC 536.42:661.426

An iterational algorithm for calculation of evaporative cooling of water droplets and films in a cooling tower is proposed. It is found from a comparison of results of theoretical calculation and experimental data that the effective radius of the droplets is approximately equal to ~$33 \cdot 10^{-4}$ m. The contribution of film cooling is shown to constitute at least 70% of the overall temperature decrease of cooled water in the cooling tower.

Introduction. In systems of reversible water supply of thermal and nuclear power plants coolers of wet type (ponds, spray ponds, cooling towers) with cooling of circulating water due to evaporation and convective heat transfer are mainly used. Modern cooling towers with height of the order of 100 m and foundation diameter of about 70 m are very expensive and important elements of a power plant [1, 2]. Analysis of their effectiveness on the basis of a limited volume of natural measurements with different atmospheric conditions is insufficiently reliable, and therefore development of a mathematical model of cooling-tower performance is an important scientific and technical problem.

In a large natural wet cooling tower, water cooling takes place both in film flows on sheets of the pack and from falling droplets of different sizes. As our calculations show, basic cooling of circulating water in the cooling tower is carried out on sheets of the pack. Droplets in the wet cooling tower are formed during water sputtering by sprinklers (nozzles) and after water runoff from the sheets of the pack to the water-collecting pond. The contribution of droplet cooling to the thermal balance of the cooling tower depends mainly on the radius of the droplets.

As seen from Fig.1, there are two zones of droplet cooling in the cooling tower: the first between the water-distribution system and the pack (spray zone), the second between the pack and the water-collecting pond (rain zone). Interference takes place between the heat and mass transfer processes in the spray zone and on the sheets of the pack. With heating of the inlet air and its saturation by water vapor while rising through the zone of film flow, the effectiveness of evaporative cooling of water droplets in the spray zone decreases. On the other hand, additional heating of the air in the spray zone increases the velocity of convective flows of air in the cooling tower and, in consequence, increases the intensity of evaporative cooling of the film flows. For the steady-state regime of cooling-tower performance, simulation of joint cooling of water droplets and films enables one to calculate with sufficient accuracy the evaporative cooling of water droplets in the spray zone and films in the pack with account for the interference, as well as to determine the thermal efficiency of the cooling tower on the whole. The use of empirical information becomes practically minimal. The effective radius of the droplets in the spray zone of the cooling tower is the only fitting parameter.

We note that the processes of heat and mass transfer from the droplet surface in cooling towers are studied in a number of experimental works. There is substantial discrepancy between the results of these

Academic Scientific Complex "A. V. Luikov Heat and Mass Transfer Institute," National Academy of Sciences of Belarus, Minsk, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 74, No. 1, pp. 45–49, January–February, 2001. Original article submitted June 8, 2000.

1062-0125/01/7401-0062$25.00

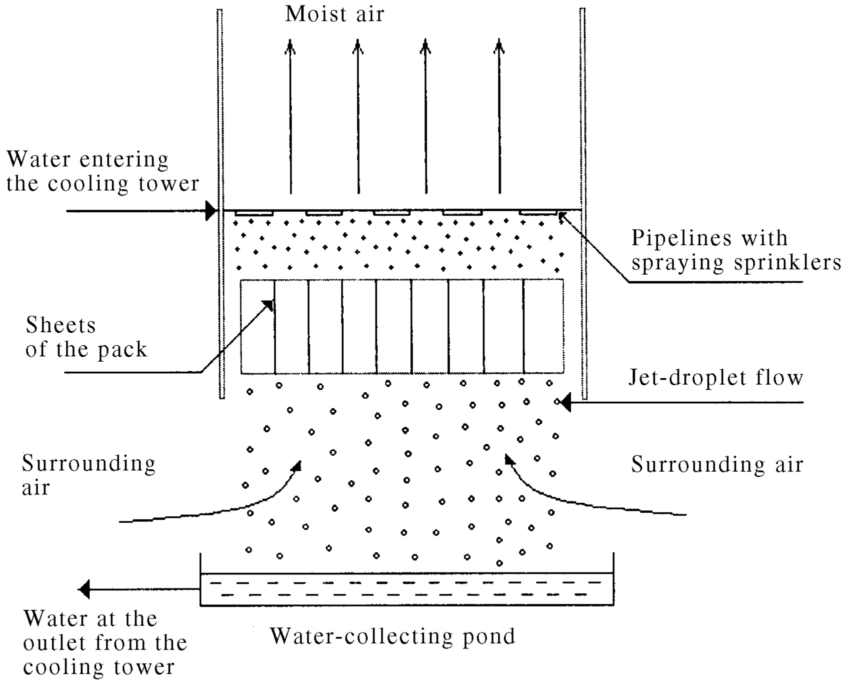

Fig. 1. Scheme of the zones of heat and mass transfer in a large natural cooling tower.

works. This fact seems to be explained by a difference in the techniques of measurements. For example, it was noted in [3] that a quite insignificant quantity of heat is removed under free fall of droplets in the rain zone, which enables the authors to consider the rain zone of existing cooling towers as a "dead zone" from the point of view of heat and mass transfer effectiveness. At the same time, the opposite point of view is presented in [4], where it is noted that a substantial quantity of heat is removed from falling water droplets by rising cold air under identical conditions of cooling. Our calculations [5] and visual observations confirm that rather large droplets form in the rain zone of a cooling tower and the above-mentioned conclusion from [3] is valid. A mathematical model of evaporative cooling of water films in a cooling tower is presented in [6], and results of its comparison with our experimental data are given in [7]. It follows from this comparison that the contribution of processes of heat and mass transfer of water droplets with upward flow of moist air cannot be neglected for rather large densities of the water flow rate per unit area of the pack.

We recall that the limiting temperature T_{lim} of evaporative cooling of water can be obtained from the condition [2]

$$\rho_s(T_{\text{lim}}) = \psi\, \rho_s(T_a)\,. \tag{1}$$

Mathematical Model of Heat and Mass Transfer of Droplets. We direct the z axis upward vertically and link the origin of coordinates with the upper edge of the pack sheets. The droplet moves under gravity and the force of aerodynamic drag arising due to the upward air flow. The temperature and humidity of the moist air flow increase as the flow rises. To calculate the characteristics of the heat and mass transfer processes for accelerating a droplet in the upward air flow, we use a system of differential equations that includes:

an equation describing the change in the droplet radius $R(z)$ due to evaporation

$$\frac{dR(z)}{dz} = -\frac{\gamma(\text{Re})\,[\rho_s(T(z)) - \rho(z)]}{\rho_w v(z)}\,; \tag{2}$$

an equation determining the change in the velocity $v(z)$ of the falling droplet

$$\frac{dv\,(z)}{dz} = \frac{g}{v\,(z)} - C\,(\mathrm{Re})\,\frac{\rho_\mathrm{a}\,[v\,(z) - u]^2}{2v\,(z)}\,\frac{\pi R^2\,(z)}{m}\,; \tag{3}$$

an equation for determination of the droplet temperature $T(z)$ averaged over its volume

$$\frac{dT\,(z)}{dz} = \frac{3\alpha\,(\mathrm{Re})\,[T_\mathrm{a}\,(z) - T\,(z)]}{c_\mathrm{w}\,\rho_\mathrm{w} R\,(z)\,v\,(z)} + \frac{3\gamma\,(\mathrm{Re})}{c_\mathrm{w}\,\rho_\mathrm{w} R\,(z)\,v\,(z)}\,[\rho_\mathrm{s}\,(T\,(z)) -$$

$$- \rho\,(z)]\,(r - c_\mathrm{w} T\,(z))\,; \tag{4}$$

an equation for calculation of the temperature of the moist air $T_\mathrm{a}(z)$

$$\frac{dT_\mathrm{a}\,(z)}{dz} = \frac{4\pi R^2\,(z)\,N_\mathrm{d}}{\rho_\mathrm{a} c_\mathrm{a}\,(v\,(z) - u)}\,[\alpha\,(\mathrm{Re})\,[T_\mathrm{a}\,(z) - T\,(z)]]\,; \tag{5}$$

an equation for the change in the density of the water vapor $\rho(z)$

$$\frac{d\rho\,(z)}{dz} - \frac{4\pi R^2\,(z)\,N_\mathrm{d}}{v\,(z) - u}\,\gamma\,(\mathrm{Re})\,[\rho_\mathrm{s}\,(T\,(z)) - \rho\,(z)]\,. \tag{6}$$

For droplets in the spray zone, the boundary conditions are written in the following way. Initial values are defined at $z = H$ for

the droplet radius

$$R|_{z=H} = R_0\,, \tag{7}$$

the droplet temperature

$$T|_{z=H} = T_1\,, \tag{8}$$

the droplet velocity

$$v|_{z=H} = v_0\,. \tag{9}$$

The following values are defined at $z = 0$, which corresponds to the upper level of the sheets of the cooling tower:

the air temperature at the outlet from the pack

$$T_\mathrm{a}|_{z=0} = T_2\,, \tag{10}$$

the density of the water vapor in the air at the outlet from the pack

$$\rho|_{z=0} = \rho_2\,. \tag{11}$$

The system of ordinary differential equations (2)–(6) and the boundary conditions (7)–(11) represent a nonlinear boundary-value problem. We emphasize that for our model the influence of the droplets on the parameters of the moist air is directly proportional to the number of droplets per unit volume or, in other words, is directly proportional to the flow rate of the water. With low flow rates of the water in the cooling tower, it is possible to neglect the effects of evaporative cooling of the droplets, which agrees with the conclusions of [7].

In accordance with [8], the heat transfer coefficient of the droplets in the air medium $\alpha(\mathrm{Re})$ occurring in Eqs. (4)–(5) was determined from the criterial dependence

$$\mathrm{Nu} = 2 + 0.5\,\mathrm{Re}^{0.5}\,, \tag{12}$$

where the Nusselt number $\mathrm{Nu} = 2R\alpha(\mathrm{Re})/\lambda$, λ is the thermal conductivity of the air, and the Reynolds number $\mathrm{Re} = 2\rho_a R(z)|v(z) - u|\mu_a$.

The mass transfer coefficient of the droplets was defined from the correlation

$$\mathrm{Sh} = 2 + 0.5\mathrm{Re}^{0.5}\,, \tag{13}$$

where the Sherwood number $\mathrm{Sh} = 2R\gamma(\mathrm{Re})/D$, and D is the diffusion coefficient of the water vapor in the air.

The coefficient C of aerodynamic drag of a droplet was calculated by the formula

$$C = \frac{24}{\mathrm{Re}}\left(1 + \frac{1}{6}\,\mathrm{Re}^{2/3}\right).$$

The vertical component u of the air velocity in the spray zone of the large cooling tower is described by the following expression:

$$u = \left(\frac{R_1}{R_2}\right)^2 \left[\frac{2gH_t\,\Delta\rho_m}{\rho_m}\right]^{0.5} k\,. \tag{14}$$

In our calculations the empirical coefficient k is equal to 0.5 [9]. In expression (14) the difference between the densities of the moist air inside and outside the cooling tower, $\Delta\rho_m$, is determined by the change in the air temperature and humidity above the pack. A contribution to $\Delta\rho_m$ is given by the heat and mass transfer of both the droplets and the flowing films. Solution of the boundary-value problem was carried out by the "shooting" method [10]. For numerical solution of the system of differential equations, the Runge–Kutta method of fourth order with a fixed step was used. Control of the accuracy was realized by means of the residual criterion Σ:

$$\Sigma = \sqrt{\left(\frac{T_a(H) - T_2}{T_2}\right)^2 + \left(\frac{\rho(H) - \rho}{\rho}\right)^2}\,.$$

Solution of the problem ended with fulfillment of the condition $\Sigma < 10^{-2}$. A further increase in the accuracy of the calculations practically did not affect the numerical results. Previously, we developed a mathematical model of evaporative cooling of water films [6].

For simulation of the steady-state process of evaporative cooling of water droplets and films in the cooling tower, an iterative algorithm was used. At first, with an assigned flow rate and temperature of the entering water, cooling of water films was calculated without taking account of the contribution of the heat and mass transfer at the droplets. As a result of calculation in accordance with the film model presented in [6], the velocity, temperature, and density of the water vapor in the moist-air flow going out of the pack were found. Then these data were used for calculation of the evaporative cooling of water droplets in the spray zone.

The average initial radius of the droplets R_0 is the fitting parameter in our model (since it is determined in many respects by the design of the sprinklers of the cooling tower). The specific flow rate of the water Q_w and the number of water droplets per unit volume N_d are related by the expression

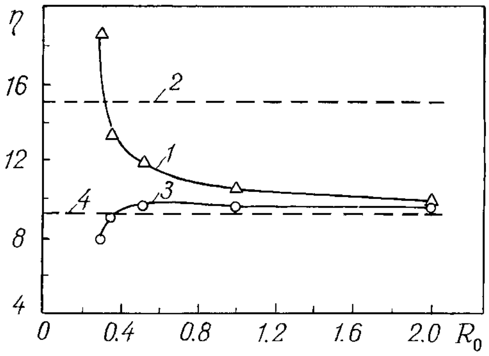

Fig. 2. Dependences of the thermal efficiency of the cooling tower η on the initial radius of the droplets R_0 for T_{w0} = 29.4°C, T_{a0} = 25.8°C, ψ = 41.8%, Q_w = 1.48 kg/(m^2·sec): 1) calculated value; 2) experimental value; 3, 4) values for film cooling with and without account for the droplets above the pack, respectively. η, %; R_0, mm.

$$N_d = \frac{Q_w}{m v_d} .$$

For a path of fall of about 1 m and a velocity of upward flow of about 1 m/sec, as our calculations showed, small changes in the initial velocity of the droplets practically do not affect the value of the final temperature. With account for the additional heating of the air in droplet cooling, the new free convection velocity was determined. For this velocity, the evaporative cooling of the water films was calculated again. The final temperature of the water droplets was the initial one for the films. For the flow going out of the pack of the cooling tower, new values of the velocity, temperature, and density of the water vapor were found on the basis of the results of calculation of the film flow. For the initial radius of the droplets the evaporative cooling of the droplets and the following temperature of the water films were calculated again. The procedure was repeated until the thermal efficiency of the cooling tower stopped changing as a result of the iterations.

As in our previous works [5–7, 9] we will describe the effectiveness of the evaporative cooling of the water by means of the thermal efficiency of the cooling tower η:

$$\eta = \frac{T_{in} - T_{out}}{T_{in} - T_{lim}} .$$

Results of the Simulation. Our mathematical model enables one to calculate the temperature of the droplets, the density of the water vapor, and the temperature of the moist air as functions of the height of droplet fall. We do not present them, but note that all the parameters change in a small region at the end of droplet fall, i.e., in the meeting of the droplets with relatively cold moist air. From the point of view of analysis of the thermal efficiency of the cooling tower, only integral effects related to evaporative cooling of the droplets will be presented below. Figure 2 shows the thermal efficiency versus the initial droplet radius above the pack.

It is easy to find from the intersection of curves 1 and 2, that is, from coincidence of the calculated and experimental values of the thermal efficiency, that the effective radius of a droplet R_{eff} is equal to $3 \cdot 10^{-4}$ m for such conditions of cooling tower performance. As our calculations showed, the contribution of evaporative cooling of water droplets with radius $R \sim 2$ mm does not exceed 2% of the overall efficiency of the cooling tower. It is of interest to note that for large hydraulic loads the effective radius of a droplet is also equal to $R_{eff} = 3 \cdot 10^{-4}$ m with a relative error of 5%. The water temperature was the same in these experiments.

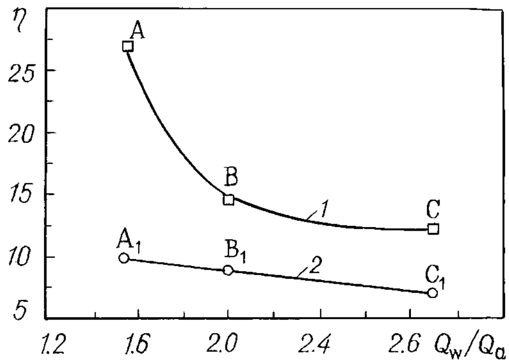

Fig. 3. Calculated values of the thermal efficiency of the cooling tower (1) and film flow (2) as functions of the ratio of the mass flow rates of the water and the air. Points A and A_1, B and B_1, and C and C_1 correspond to a temperature of the entering water of 34.4, 29.4, and 26.9°C, respectively.

The contribution of film cooling with joint cooling of the water droplets and films (curve 3) was calculated from the formula

$$\frac{T_{\text{in}} - \Delta T_{\text{d}} - T_{\text{out}}}{T_{\text{in}} - T_{\text{lim}}}.$$

The temperature of the water affects the effective radius of the droplets via a change in the surface-tension coefficient. For the same flow rate of the water, we can assume with good accuracy that the effective radius of a droplet is directly proportional to the value of the surface-tension coefficient [11, 12]. In particular, it follows that for a fixed flow rate and the conditions presented in Fig. 2, an increase in the water temperature by 5°C decreases the droplet radius by ~1%. According to Fig. 2, even a slight decrease in the droplet radius near R_{eff} leads to a substantial increase in the thermal efficiency of the evaporative cooling of the droplets.

Figure 3 enables one to compare the calculated thermal efficiency of the cooling tower and film cooling on the sheets of the pack as a function of the ratio of the mass flow rates of the water and the air. We emphasize that the flow rate of the water is the same for all the cases presented. It is seen that with increase in the temperature of the entering water the efficiency of the cooling tower increases mainly due to droplet cooling. This effect can be explained in the following way. First, the increase in the initial temperature of the water leads automatically to an increase in the air draft in the cooling tower. Second, with increase in this temperature the effective radius of the droplets decreases, resulting in an increase in the efficiency of the droplet cooling. This conclusion agrees satisfactorily with the result of treatment of experimental data on the dependence of the thermal efficiency of an industrial cooling tower [7] on the ratio of the mass flow rates of the water and the air.

Conclusions. In the spray zone, as calculations according to our nonlinear mathematical model showed, only water droplets with rather small radii influence substantially the efficiency of the evaporative cooling. This influence depends on the velocity of the upward moist-air flow. In turn, this velocity depends on the intensity of the evaporative cooling in the zone of film flow in the cooling tower.

A comparison of the simulation results and experimental data on the thermal efficiency of a cooling tower showed that the effective radius of the droplets was approximately identical for all hydraulic loads. It follows that the number of droplets per unit volume is directly proportional to the water flow rate. Therefore, the surface of heat and mass transfer of the droplet flow is also directly proportional to the water flow rate.

That is why the deviation of simulation results for a purely film flow from experimental ones should increase with increasing water flow rate. This conclusion is confirmed by a comparison [7] of simulation results and experimental data. It follows from treatment of experimental data on the thermal efficiency of a cooling tower that the effective radius of the droplets is approximately equal to $3 \cdot 10^{-4}$ m. The contribution of evaporative film cooling calculated with account for the heat and mass transfer at the droplets is at least 70%. Additional heating of the air due to evaporative cooling of the water droplets leads to an increase in the draft of the cooling tower, which in turn increases the efficiency of film cooling.

NOTATION

ρ, density of the water vapor; ρ_s, density of the saturated vapor; ρ_w, density of the water; ρ_a, density of the inlet air in the cooling tower; $T(z)$, temperature of a droplet; T_a, temperature of the surrounding air; T_{lim}, limiting temperature of cooling of the water in evaporative cooling; T_{in}, initial temperature of the water; T_{out}, final temperature of the water; ΔT_d, change in the temperature of the droplets in the spray zone; ψ, relative humidity of the air; $R(z)$, radius of a droplet, mm; R_{eff}, effective radius of a droplet, mm; R_0, initial radius of a droplet, mm; R_1 and R_2, radii of the orifice and foundation of the cooling tower, respectively; H, height of the plume of water; H_t, height of the cooling tower from the pack to the orifice; $v(z)$, velocity of a droplet; v_d, initial velocity of a droplet; u, velocity of the rising moist air above the pack; γ(Re), mass-transfer coefficient; α(Re), heat-transfer coefficient; C, coefficient of aerodynamic drag; g, gravitational acceleration; m, mass of a droplet; N_d, number of droplets in unit volume; c_w, heat capacity of the water; c_a, heat capacity of the surrounding air; μ_a, coefficient of dynamic viscosity of the air; r, latent heat of vaporization; k, empirical coefficient; Σ, criterion of the residual; Q_w, specific hydraulic load of the water; η, thermal efficiency. Subscripts: s, saturated; a, air; lim, limit; w, water; d, droplet; m, mixture; t, tower; in, water at the inlet; out, water at the outlet.

REFERENCES

1. B. S. Farforovskii and V. B. Farforovskii, *Coolers of Circulating Water of Thermal Power Plants* [in Russian], Leningrad (1972).
2. L. D. Berman, *Evaporative Cooling of Circulating Water*, Pergamon Press, Oxford (1961).
3. N. V. Barsukov, in: *Proc. of an Interinstitutional Scientific Seminar on Problems of Power Engineering*, Balakovo, October 25–28, 1994 [in Russian], Saratov (1996), pp. 17–18.
4. J. H. Mercker, A. A. Dreyer, and P. J. Erens, in: *Proc. 10th Int. Heat Transfer Conf.*, Vol. 3, Brighton (1994), pp. 305–310.
5. S. P. Fisenko, *Inzh.-Fiz. Zh.*, **64**, No. 2, 154–159 (1993).
6. A. I. Petruchik and S. P. Fisenko, *Inzh.-Fiz. Zh.*, **72**, No. 1, 43–49 (1999).
7. V. V. Antonik, A. I. Petruchik, A. D. Solodukhin, N. N. Stolovich, and S. P. Fisenko, in: *Proc. IV Minsk Int. Forum "Heat and Mass Transfer–MIF-2000,*" Minsk, May 22–26, 2000 [in Russian], Vol. 5, Minsk (2000), pp. 314–324.
8. A. I. Ivandaev, A. G. Kutushev, and R. I. Nigmatulin, *Advances in Science and Technology, Mechanics of Fluid and Gas* [in Russian], Moscow (1987).
9. A. V. Vlasov, E. O. Voronov, G. V. Dashkov, A. D. Solodukhin, and S. P. Fisenko, in: *Heat and Mass Transfer-95* [in Russian], Minsk (1995), pp. 23–27.
10. N. N. Kalitkin, *Numerical Methods* [in Russian], Moscow (1978).
11. V. G. Levich, *Physicochemical Hydrodynamics*, Prentice-Hall, New York (1962).
12. A. M. Kutepov, A. D. Polyanin, Z. D. Zapryanov, A. V. Vyaz'min, and D. A. Kazenin, *Chemical Hydrodynamics* [in Russian], Moscow (1996).